Center-of-mass effects in electromagnetic two-proton knockout reactions

Carlotta Giusti,[1] Franco Davide Pacati,[1] Michael Schwamb,[2,3] and Sigfrido Boffi[1]

[1] *Dipartimento di Fisica Nucleare e Teorica, Università degli Studi di Pavia Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy*

[2] *Dipartimento di Fisica, Università degli Studi di Trento and Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Trento, I-38100 Povo (Trento), Italy*

[3] *European Center for Theoretical Studies in Nuclear Physics and Related Areas (ECT*), I-38100 Villazzano (Trento), Italy*

The role of center-of mass (CM) effects in the one-body nuclear current in the description of electromagnetically induced two-nucleon knockout reactions is discussed in connection with the problem of the lack of orthogonality between initial bound states and final scattering states obtained by the use of an energy-dependent optical model potential. Results for the cross sections of the exclusive ^{16}O(e,e$'$pp)^{14}C and ^{16}O(γ,pp)^{14}C knockout reactions in different kinematics are presented and discussed. In super-parallel kinematics CM effects produce a strong enhancement of the ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ cross section which strongly reduces the destructive interference between the one-body and Δ-current and the sensitivity to the treatment of the Δ-current found in previous work.

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I. INTRODUCTION

The investigation of nuclear structure is one of the most important and ambitious aims of hadronic physics. A reasonable starting point is offered by the independent particle shell model. However, the incorporation of additional short-range correlations (SRC) beyond a mean-field description turns out to be inevitably necessary for a proper description of nuclear binding. The most direct reaction to study SRC is naturally electromagnetically induced two-nucleon knockout. Intuitively, the probability that a photon is absorbed by a nucleon pair should be a direct measure for SRC. However, due to competing two-body effects like meson-exchange currents (MEC) or final state interactions (FSI), this simple picture needs to be modified in order to obtain quantitative predictions. Ideally, the role of MEC and FSI should be small or at least under control in order to extract information on SRC from experiment. This requires a theoretical approach which should be as comprehensive as possible. An overview over the available theoretical models till the middle of the 90s can be found in [1]. Presently, different models are available (see [2–4] and references therein). Due to the conceptual complexity of the nuclear many-body problem, various approximations and simplifying assumptions are needed for practical calculations. Thus, usually different treatments of initial bound and final scattering states are adopted in the models.

In the Pavia model [4, 5] bound and scattering states are, in principle, consistently derived as eigenfunctions of an energy-dependent non-Hermitian Feshbach-type optical potential. However, in actual calculations the initial hadronic state is obtained from a recent calculation of the two-nucleon spectral function [6] where different types of correlations are included consistently. For the final hadronic state, a complex phenomenological optical potential, derived through a fit to nucleon-nucleus scattering data, is used for the description of the FSI between the outgoing nucleons and the residual nucleus. The mutual nucleon-nucleon interaction (NN-FSI) in the final state can be taken into account at least perturbatively [7, 8].

Independently of the specific prescriptions adopted in the calculations, a conceptual problem arises in the model where the initial and final states, which are eigenfunctions of an energy-dependent optical potential at different energies, are, as such, not orthogonal. Indeed, the process involves transitions between bound and continuum states which must be orthogonal, since they are eigenfunctions of the full nuclear many-body Hamiltionian at different energies. Orthogonality is in general lost in a model when the descripion is restricted to a subspace where other channels are suppressed. The description of direct knockout reactions in terms of the eigenfunctions of a complex energy-dependent optical potential considers only partially the contribution of competing inelastic channels. The remaining effects due to occuring inelasticities can, in principle, be taken into account by a suitable effective transition operator, which removes the orthogonality defect of the model wave functions [9]. In practice, however, the usual approach does not make use of an effective operator.

The present paper deals with the proper treatment of all the CM effects in the matrix elements of the one-body nuclear current in connection with the problem of the lack of orthogonality between initial and final states in the calculation of the cross section of the electromagnetic two-nucleon knockout reactions.

The reaction mechanism and CM effects are discussed in sect. II. Different prescriptions are proposed to cure the spuriosity which may result in the numerical calculations as a consequence of the orthogonality defect. These prescriptions are discussed and related to a proper treatment of all the CM effects in the transition matrix elements. In sect. III the effects of CM and orthogonality are illustrated, with specific numerical examples in selected kinematics, for the exclusive ^{16}O(e,e$'$pp)^{14}C and ^{16}O(γ,pp)^{14}C knockout reactions. A summary and some conclusions can be found in sect. IV.

II. REACTION MECHANISM AND CENTER-OF-MASS EFFECTS

The basic ingredients for the calculation of the cross section of the reaction induced by a real or virtual photon, with momentum $\boldsymbol{q}$, where two nucleons, with momenta $\boldsymbol{p}_1'$, and $\boldsymbol{p}_2'$, are ejected from a nucleus, are given by the transition matrix elements of the charge-current density operator between initial and final nuclear states

$$J^\mu(\boldsymbol{q}) = \int < \Psi_{\mathrm{f}}|\hat{J}^\mu(\boldsymbol{r})|\Psi_{\mathrm{i}} > \mathrm{e}^{\,\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r}. \tag{1}$$

Bilinear products of these integrals give the components of the hadron tensor, whose suitable combinations allow the calculation of all the observables available from the reaction process [1, 5].

If the residual nucleus is left in a discrete eigenstate of its Hamiltonian, i.e. for an exclusive process, and under the assumption of the direct knock-out mechanism, the matrix elements of Eq. (1) can be written as [5, 10][1]

$$J^\mu(\boldsymbol{q}) = \int \widetilde{\psi}_{\mathrm{f}}^*(\boldsymbol{r}_1,\boldsymbol{r}_2) J^\mu(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2)\widetilde{\psi}_{\mathrm{i}}(\boldsymbol{r}_1,\boldsymbol{r}_2)\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2. \tag{2}$$

Eq. (2) contains three main ingredients: the two-nucleon scattering wave function $\widetilde{\psi}_{\mathrm{f}}$, the nuclear current J^μ and the two-nucleon overlap integral (TOF) $\widetilde{\psi}_{\mathrm{i}}$ between the ground state of the target and the final state of the residual nucleus.

The nuclear current J^μ is the sum of a one-body and a two-body contribution, i.e.

$$J^\mu(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) = J^{(1)\mu}(\boldsymbol{r},\boldsymbol{r}_1) + J^{(1)\mu}(\boldsymbol{r},\boldsymbol{r}_2) + J^{(2)\mu}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2). \tag{3}$$

The one-body (OB) part includes the longitudinal charge term and the transverse convective and spin currents, and can be written as

$$J^{(1)\mu}(\boldsymbol{r},\boldsymbol{r}_k) = j^{(1)\mu}(\boldsymbol{r},\boldsymbol{\sigma}_k)\,\delta(\boldsymbol{r}-\boldsymbol{r}_k) \tag{4}$$

with $k = 1, 2$. The two-body current is derived from the effective Lagrangian of [11], performing a non relativistic reduction of the lowest-order Feynman diagrams with one-pion exchange. We have thus currents corresponding to the seagull and pion-in-flight diagrams, and to the diagrams with intermediate Δ-isobar configurations [12], i.e.

$$\boldsymbol{J}^{(2)}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) = \boldsymbol{J}^{\mathrm{sea}}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) + \ \boldsymbol{J}^{\pi}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) + \boldsymbol{J}^{\Delta}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2). \tag{5}$$

For two-proton emission the seagull and pion-in-flight meson-exchange currents and the charge-exchange contribution of the Δ-current are vanishing in the nonrelativistic limit. The surviving components of the Δ-current can be written as

$$J^{(2)\mu}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) = j^{(2)\mu}(\boldsymbol{r}_{12},\boldsymbol{\sigma}_2,\boldsymbol{\tau}_2)\,\delta(\boldsymbol{r}-\boldsymbol{r}_1) + j^{(2)\mu}(\boldsymbol{r}_{12},\boldsymbol{\sigma}_1,\boldsymbol{\tau}_1)\,\delta(\boldsymbol{r}-\boldsymbol{r}_2). \tag{6}$$

with $\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2$. Details of the nuclear current components can be found in [4, 12–14]. More specifically, the various treatments and parametrizations of the Δ-current used in the calculations are given in [4].

In order to evaluate the transition amplitude of Eq. (2), for the three-body system consisting of the two protons, 1 and 2, and of the residual nucleus B, it appears to be natural to work with CM coordinates [5, 15]

$$\begin{aligned} \boldsymbol{r}_{1B} &= \boldsymbol{r}_1 - \boldsymbol{r}_B,\ \boldsymbol{r}_{2B} = \boldsymbol{r}_2 - \boldsymbol{r}_B, \\ \boldsymbol{r}_B &= \sum_{i=3}^{A} \boldsymbol{r}_i/(A-2). \end{aligned} \tag{7}$$

[1] Spin/isospin indices are generally suppressed in the formulas of this paper for the sake of simplicity.

The conjugated momenta are given by

$$\boldsymbol{p}_{1B} = \frac{A-1}{A}\boldsymbol{p'}_1 - \frac{1}{A}\boldsymbol{p'}_2 - \frac{1}{A}\boldsymbol{p}_B \ , \tag{8}$$

$$\boldsymbol{p}_{2B} = -\frac{1}{A}\boldsymbol{p'}_1 + \frac{A-1}{A}\boldsymbol{p'}_2 - \frac{1}{A}\boldsymbol{p}_B \ , \tag{9}$$

$$\boldsymbol{P} = \boldsymbol{p'}_1 + \boldsymbol{p'}_2 + \boldsymbol{p}_B \ , \tag{10}$$

where $\boldsymbol{p}_B = \boldsymbol{q} - \boldsymbol{p}'_1 - \boldsymbol{p}'_2$ is the momentum of the residual nucleus in the laboratory frame.

With the help of these relations, one can cast the transition amplitude (2) into the following form

$$J^\mu(\boldsymbol{q}) = \int \psi^*_{\mathrm{f}}(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) V^\mu(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) \psi_{\mathrm{i}}(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) \mathrm{d}\boldsymbol{r}_{1B} \mathrm{d}\boldsymbol{r}_{2B}, \tag{11}$$

with the definition

$$\psi_{\mathrm{i/f}}(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) := \widetilde{\psi}_{\mathrm{i/f}}(\boldsymbol{r}_1, \boldsymbol{r}_2) \tag{12}$$

and the expression

$$V^\mu(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) = \exp\left(i\boldsymbol{q}\frac{A-1}{A}\boldsymbol{r}_{1B}\right) \exp\left(-i\boldsymbol{q}\frac{1}{A}\boldsymbol{r}_{2B}\right) \left(j^{(1)\mu}(\boldsymbol{r}_{1B}, \boldsymbol{\sigma}_1) + j^{(2)\mu}(\boldsymbol{r}_{12}, \boldsymbol{\sigma}_2, \boldsymbol{\tau}_2)\right) \quad + (1 \leftrightarrow 2) \ . \tag{13}$$

It is generally thought that the contribution of the OB current to two-nucleon knockout is entirely due to the correlations included in the two-nucleon wave function. In fact, a OB operator cannot affect two particles if they are not correlated. It can be seen, however, from Eq. (13) that in the CM frame the transition operator becomes a two-body operator even in the case of a OB nuclear current. Only in the limit $A \to \infty$ CM effects are neglected and the expression in Eq. (11) vanishes for a pure OB current in Eq. (13) sandwiched between orthogonalized single particle (s.p.) wave functions. This means that, due to this CM effect, for finite nuclei the OB current can give a contribution to the cross section of two-particle emission independently of correlations. This effect is similar to the one of the effective charges in electromagnetic reactions [16].

The matrix elements of Eq. (11) involve bound and scattering states, ψ_{i} and ψ_{f}, which are consistently derived from an energy-dependent non-Hermitian Feshbach-type Hamiltonian for the considered final state of the residual nucleus. They are eigenfunctions of this Hamiltonian at negative and positive energy values [1, 5]. However, in practice, it is not possible to achieve this consistency and the treatment of initial and final states proceeds separately with different approximations.

The two-nucleon overlap function (TOF) ψ_{i} contains information on nuclear structure and correlations. Different approaches are used in [6, 10, 17, 18]. In the present calculations the TOF is obtained as in [6], from the most recent calculation of the two-proton spectral function of ^{16}O, where both SRC and long-range correlations are included consistently with a two-step procedure.

In the calculation of the final-state wave function ψ_{f} only the interaction of each one of the two outgoing nucleons with the residual nucleus is included. Therefore, the scattering state is written as the product of two uncoupled s.p. distorted wave functions, eigenfunctions of a complex phenomenological optical potential which contains a central, a Coulomb, and a spin-orbit term [19]. The effect of the mutual interaction between the two outgoing nucleons has been studied in [7, 8, 20] and can in principle be included as in [7, 8].

The matrix element of Eq. (11) contains a spurious contribution since it does not vanish when the transition operator V is set equal to 1. This is essentially due to the lack of orthogonality between the initial and the final state wave functions. In the model the use of an effective nuclear current operator removes the orthogonality defect besides taking into account space truncation effects [1, 9]. In the usual approach of Eq. (11), however, the effective operator is replaced by the bare nuclear current operator. Thus, it is this replacement which may introduce a spurious contribution which is not specifically due to the different prescriptions adopted in practical calculations, but is already present in Eq. (11), where ψ_{i} and ψ_{f} are eigenfunctions of an energy-dependent Feshbach-type Hamiltonian at different energies. In the past this spuriousity was cured by subtracting from the transition amplitude the contribution of the OB current without correlations in the nuclear wave functions. In detail, the expression

$$\int \psi^*_{\mathrm{f}}(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) \left(\exp\left(i\boldsymbol{q}\frac{A-1}{A}\boldsymbol{r}_{1B}\right) \exp\left(-i\boldsymbol{q}\frac{1}{A}\boldsymbol{r}_{2B}\right) j^{(1)\mu}(\boldsymbol{r}_{1B}, \boldsymbol{\sigma}_1) + 1 \leftrightarrow 2 \right) \psi_{\mathrm{i,no\,Cor}}(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) \mathrm{d}\boldsymbol{r}_{1B} \mathrm{d}\boldsymbol{r}_{2B} \tag{14}$$

was subtracted from (11), where in the initial state $\psi_{\mathrm{i,no\,Cor}}$ SRC are ignored. This prescription is denoted as approach A in the proceeding discussions. In this approach, however, we do not subtract only the spuriosity, but also the CM effect given by the two-body operator in Eq. (13), which is present in the OB current independently of correlations and which is not spurious. The relevance of this effect can be estimated comparing our previous results with the results of a different prescription, that is denoted as approach B in the proceeding discussions, where we subtract from (11) instead of (14) the spurious contribution due to the OB current without correlations and without CM corrections. This can be achieved by putting the limit $A \to \infty$ in (14), i.e. by the expression

$$\int \psi_{\mathrm{f}}^{*}(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) \left(\exp\left(i\boldsymbol{q}\boldsymbol{r}_{1B}\right) j^{(1)\mu}(\boldsymbol{r}_{1B}, \boldsymbol{\sigma}_1) + 1 \leftrightarrow 2 \right) \psi_{\mathrm{i,no\,Cor}}(\boldsymbol{r}_{1B}, \boldsymbol{r}_{2B}) \mathrm{d}\boldsymbol{r}_{1B} \mathrm{d}\boldsymbol{r}_{2B} \ . \tag{15}$$

This prescription gives an improved, although still rough, evaluation of the spurious contribution.

An alternative and more accurate procedure to get rid of the spuriosity is to enforce orthogonality between the initial and final states by means of a Gram-Schmidt orthogonalization [21]. In this approach each one of the two s.p. distorted wave functions is orthogonalized to all the s.p. shell-model wave functions that are used to calculate the TOF, i.e., for the TOF of [6], to the h.o. states of the basis used in the calculation of the spectral function, which range from the $0s$ up to the $1p$-$0f$ shell. This more accurate procedure, that we denote as approach C in the proceeding discussions, allows us to get rid of the spurious contribution to two-nucleon emission due to a OB operator acting on either nucleon of an uncorrelated pair, which is due to the lack of orthogonality between the s.p. bound and scattering states of the pair. In this approach, in consequence, no OB current contribution without correlations like (14) or (15) needs to be subtracted. Moreover, it allows us to include automatically all CM effects via (13).

III. RESULTS

The effects of CM and orthogonalization have been investigated for the exclusive ^{16}O(e,e$'$pp)^{14}C and ^{16}O(γ,pp)^{14}C reactions.

Calculations performed in different situations indicate that the results depend on kinematics and on the prescriptions adopted to treat the theoretical ingredients of the model. The contribution due to the CM effects in the OB current without correlations, that were neglected in our previous calculations, are in general non negligible. Although in many situations this contribution is small and does not change significantly the results, there are also situations where it is large and produces important quantitative and qualitative differences. This is the case of the super-parallel kinematics, where these effects are maximized. The super-parallel kinematics is therefore of particular interest for our study.

In the so-called super-parallel kinematics the two nucleons are ejected parallel and anti-parallel to the momentum transfer and, for a fixed value of the energy ω and momentum transfer q, it is possible to explore, for different values of the kinetic energies of the outgoing nucleons, all possible values of the recoil momentum p_B. This kinematical setting has been widely investigated in our previous work [4–8, 10, 17] and is of particular interest from the experimental point of view, since it has been realized in the recent ^{16}O(e,e$'$pp)^{14}C [22] and ^{16}O(e,e$'$pn)^{14}N [23] experiments at MAMI. The super-parallel kinematics chosen for the present calculations of the ^{16}O(e,e$'$pp)^{14}C reaction is the same already considered in our previous work and realized in the experiment [22] at MAMI, i.e. the incident electron energy is $E_0 = 855$ MeV, $\omega = 215$ MeV, and $q = 316$ MeV$/c$.

The cross section of the ^{16}O(e,e$'$pp)^{14}C reaction to the 0^+ ground state of ^{14}C calculated in the super-parallel kinematics is displayed in Fig. 1 for the three different approaches A (dotted), B (dashed) and C (solid). The CM contribution included in approach B produces a large enhancement of the cross section calculated with the OB current. The results are shown in the right panel of the figure, where it can be seen that the enhancement is large for recoil momentum values up to about 300 MeV$/c$ and is a factor of about 5 in the maximum region. A similar result is obtained with orthogonalized initial and final states. In this case the OB cross section is a bit larger at low values of the recoil momentum and a bit lower at larger values of p_B.

The results depicted by the dashed and solid lines in Fig. 1 correspond to the two different procedures proposed to cure the spuriosity due to the lack orthogonality between initial and final states in the model. In the dashed line the spurious contribution is subtracted, in the solid line orthogonality between the s.p. states is restored. With respect to the previous result, shown by the dotted line, the dashed line includes all the CM effects, the solid line takes into account, in addition, also the effect due to the lack of orthogonality. It can be clearly seen from the comparison shown in the figure that the large difference between the old and the new results is mostly due to the CM effects and not to the treatment of the spuriosity or to the restoration of orthogonality between the initial and final state wave functions of the model.

The final cross sections given by the sum of the OB and the two-body Δ-currents are compared in the left panel of Fig. 1. Calculations have been performed with the so-called Δ(NN) parametrization [4] for the Δ-current, i.e. the parameters have been fixed considering the NN-scattering in the Δ-region, where a reasonable description of data is achieved with parameters similar to the ones of the full Bonn potential [24]. It was shown and explained in [4] that in the super-parallel kinematics, and for the transition to the ground state of ^{14}C, a regularized prescription for

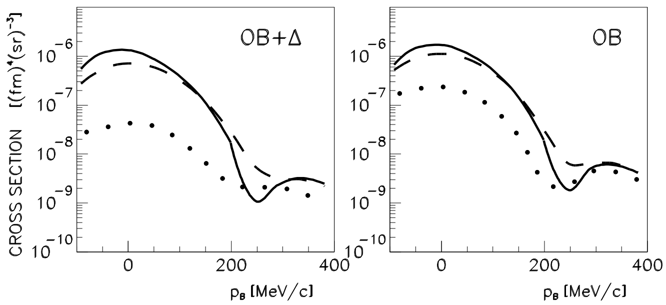

FIG. 1: The differential cross section of the ^{16}O(e,e$'$pp)^{14}C reaction to the 0^+ ground state of ^{14}C as a function of p_B in a super-parallel kinematics with $E_0 = 855$ MeV, electron scattering angle $\theta_e = 18°$, $\omega = 215$ MeV, and $q = 316$ MeV/c. Different values of p_B are obtained changing the kinetic energies of the outgoing nucleons. Positive (negative) values of p_B refer to situations where $\boldsymbol{p}_B$ is parallel (anti-parallel) to $\boldsymbol{q}$. The final results given by sum of the one-body and Δ-currents (OB+Δ) are displayed in the left panel, the separate contribution of the one-body (OB) current is shown in the right panel. The TOF from the two-proton spectral function of [6] and the Δ(NN) parametrization [4] for the Δ-current are used in the calculations. The dotted lines give the results of [4], i.e. of approach A. The dashed and solid lines refer to approach B (improved treatment of the CM contribution of the OB current) and approach C (explicit orthogonalization of s.p. bound and scattering states), respectively.

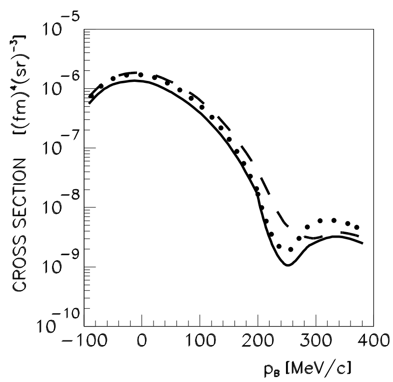

FIG. 2: The differential cross section of the ^{16}O(e,e$'$pp)^{14}C$_{\text{g.s.}}$ reaction as a function of p_B in the same super-parallel kinematics as in Fig. 1. Calculations are performed with approach C. TOF as in Fig. 1. OB (dotted line), OB+Δ(NoReg) (dashed line), OB+Δ(NN) (solid line).

the Δ, such as, e.g., Δ(NN), produces a destructive interference with the OB current which makes the final cross section lower than the OB one. This reduction is strong in our previous calculation of [4], up to about one order of magnitude. The relevance of the destructive interference depends, however, on the relative weight of the OB and Δ-current contributions [4]. The strong enhancement of the OB current contribution produced in the new calculations by CM effects reduces the destructive interference between the OB and Δ-currents. Thus, only a slight reduction of the OB current contribution is obtained in the new calculations by the addtional incorporation of the Δ-current. The final cross section is completely dominated by the OB current and at low values of the recoil momentum it is more than one order of magnitude larger than the one obtained in the old calculations. An enhancement factor of about 30 is given in the maximum region.

It was shown in [4] that dramatic differences are found in the super-parallel kinematics with different parametrizations of the Δ-current and with different TOFs.

The cross sections calculated in approach C for different Δ-parametrizations are shown in Fig. 2. The result with the regularized Δ(NN) prescription, already shown in Fig. 1, is compared with the one given by the simpler unregularized

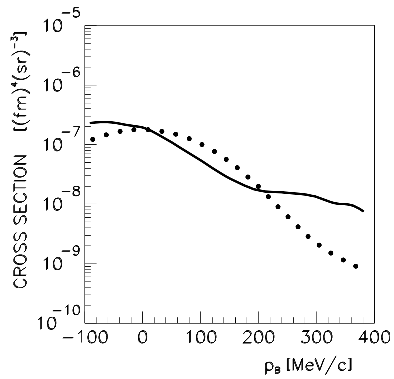

FIG. 3: The differential cross section of the ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ reaction as a function of p_B in the same super-parallel kinematics as in Fig. 1. Calculations are performed with the TOF from the simpler approach of [10] and with the Δ(NN) parametrization. The dotted line gives the result of approach A [4], the solid line is obtained with approach C.

approach Δ(NoReg) of [4]. In [4] the final cross sections calculated with these two parametrizations differ up to about one order of magnitude. Only small differences are obtained in Fig. 2. The cross section with Δ(NN) is a bit lower and the one with Δ(NoReg) a bit higher than the cross section given by the OB current.

We note that the orthogonalized wave functions are used also in the calculation of the matrix elements with the Δ-current, where the effect of orthogonalization is anyhow negligible. In practice, in the present calculations the contribution of the Δ-current is the same as in [4]. Thus, the large difference with respect to the results of [4] in Figs. 1 and 2 is due to the CM effects in the OB current and, as a consequence, to the strong reduction of the destructive interference between the OB and the Δ-current contribution calculated with the Δ(NN) parametrization.

The cross sections shown in Fig. 3 are calculated with the simpler TOF of [10], where the two-nucleon wave function is given by the product of a coupled and antisymmetrized shell model pair function and of a Jastrow-type central and state independent correlation function, taken from [25]. In this approach only SRC are considered and the final state of the residual nucleus is a pure two-hole state. The ground state of ^{14}C is a $(p_{1/2})^{-2}$ hole in ^{16}O. Thus, in the orthogonalized calculation the s.p. distorted wave functions are orthogonalized only to the $p_{1/2}$ state.

The differences between the results of approaches A and C, which are shown in Fig. 3, are significant, although less dramatic than those with the TOF from the spectral function displayed in Fig. 1, and do not change the main qualitative features of the previous results. It can be noted that in Fig. 3 the differences are larger at larger values of the recoil momentum, i.e. in the kinematical region where the differences between the corresponding results in Fig. 1 are strongly reduced.

Thus, the CM effects included in the present calculations drastically reduce the sensitivity to the treatment of the Δ-current found in [4] for the super-parallel kinematics. These CM effects are, however, very sensitive to the treatment of the TOF. The large differences given in the orthogonalized approach C by the two TOFs in Figs. 1 and 3 confirm that the cross sections are very sensitive to the treatment of correlations in the TOF. This result strongly motivates further research, both from the experimental as well as from the theoretical side, in the field of pp-knockout.

Similar calculations performed for the transition to the 1^+ excited state of ^{14}C do not show any significant difference with respect to our previous results shown in [4].

The effect of the mutual interaction between the two outgoing protons (NN-FSI) has been neglected in the calculations presented till now because it is not relevant to investigate CM effects. NN-FSI has been studied within a perturbative treatment in [7, 8], where it is found that the effect depends on the kinematics and on the type of reaction considered. Since NN-FSI turns out to be particularly strong just for the ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ reaction and in the super-parallel kinematics, it can be interesting to give here only one numerical example for this case, just to show how our previous results of [8] change in the orthogonalized approach.

The effect of the NN-FSI on the cross section of the ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ reaction in the super-parallel kinematics is shown in Fig. 4. The results obtained in the approach considered till now (DW), where only the interaction of each one of the outgoing nucleons with the residual nucleus is considered, are compared with the results of the more complete treatment (DW-NN) where also the mutual interaction between the two outgoing nucleons is included within the same perturbative approach as in [8]. The cross sections given by the separate contributions of the OB and Δ-current, as well as the ones given by the sum OB+Δ, are displayed in the figure. These results can be compared with the

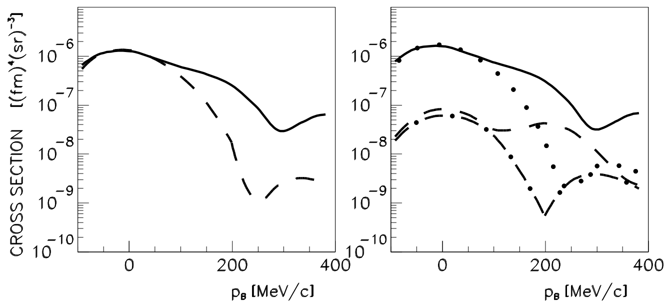

FIG. 4: The differential cross section of the ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ reaction as a function of p_B in the same super-parallel kinematics as in Fig. 1. Calculations are performed in approach C with the same TOF and Δ-parametrization as in Fig. 1. Line convention for the left panel: OB+Δ with DW-NN (solid line), OB+Δ with DW (dashed line). Line convention for the right panel: OB with DW-NN (solid line), OB with DW (dotted line), Δ-current with DW-NN (dashed line), Δ-current with DW (dot-dashed line).

corresponding ones presented in Figs. 3 and 4 of [8], which differ not only because the calculations of Fig. 4 are performed with orthogonalized initial and final states, but also because a different Δ-parametrization and a different TOF are used in the two calculations. In fact, the Δ(NN) parametrization is used in Fig. 4 compared to an old prescription of ours in [8]. The TOF of [6] is used in Fig. 4 and the one obtained from the first calculation of the spectral function of [17] in [8]. The different treatment of the various theoretical ingredients produce significant numerical differences in the calculated cross sections. The contribution of NN-FSI to the final cross section is, however, of the same type and of about the same relevance as in [8]. In particular, the considerable enhancement given by NN-FSI for medium and large values of the recoil momentum is confirmed in the present calculations. In contrast to the results of [8], where the enhancement at large p_B is due the Δ-current contribution, in Fig. 4 it is essentially due to the OB current, which is always dominant in the cross section for all the values of the recoil momentum.

A different kinematical situation is considered in Fig. 5. The ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ and ^{16}O(γ,pp)^{14}C$_{\rm g.s.}$ cross sections are calculated in a coplanar symmetrical kinematics where the two nucleons are ejected at equal energies and equal but opposite angles with respect to the momentum transfer. In this kinematical setting different values of p_B are obtained changing the scattering angles of the two outgoing protons.

The ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ cross section displayed in the top panel is calculated with $E_0 = 855$ MeV, $\theta_e = 18^\circ$, and $\omega = 215$ MeV, i.e. the same values as in the super-parallel kinematics. In this symmetrical kinematics, however, the CM effect included in the orthogonalized approach gives only small differences with respect to the previous result. The cross section is dominated by the OB current and, as in [4], it is not affected by the treatment of the Δ-current. It is, however, sensitive to the treatment of correlations in the TOF [4].

The cross section of the ^{16}O(γ,pp)^{14}C$_{\rm g.s.}$ reaction at an incident photon energy $E_\gamma = 400$ MeV, displayed in the bottom panel of Fig. 5, is dominated by the Δ-current. Thus, the effects due to CM and orthogonalization included in the present calculations, which mainly affect the OB current, do not affect the final cross section for recoil momentum values up to ~ 200 MeV/c. At higher values of p_B, where the Δ-current contribution is strongly reduced, these effects produce a large increase of the contribution of the OB current and of the final cross section. In the region where the Δ-current is dominant the sensitivity of the results to the Δ-parametrization is the same as in [4], i.e. very large.

IV. SUMMARY AND CONCLUSIONS

Two basic aspects have been discussed within the frame of electromagnetic two-proton knockout reactions, i.e. CM effects and the spuriosity arising from the lacking orthogonality between initial and final state wave functions in connection with the usual treatments of the nuclear current. They have been investigated for the cross sections of the exclusive ^{16}O(e,e$'$pp)^{14}C and ^{16}O(γ,pp)^{14}C reactions under the traditional conditions of super-parallel and symmetrical kinematics. Different kinematics and transitions to discrete low-lying states of the residual nucleus are known to emphasize either the role of the one-body currents, and thus of correlations, or of the two-body Δ-current. Since in two-nucleon knockout one is primarily interested in studying correlations, it is important to keep all the ingredients of the cross section under control in order to extract the useful information from data.

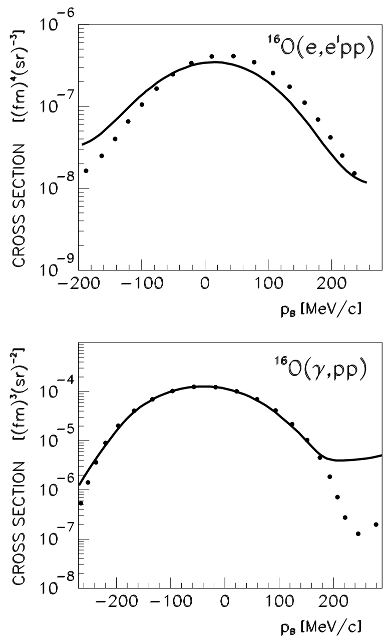

FIG. 5: The differential cross section of the ^{16}O(e,e′pp)^{14}C$_{\rm g.s.}$ (top panel) and ^{16}O(γ,pp)^{14}C$_{\rm g.s.}$ (bottom panel) reactions as a function of p_B in a coplanar symmetrical kinematics with $E_0 = 855$ MeV, $\theta_e = 18^\circ$, $\omega = 215$ MeV, and $q = 316$ MeV/c (top panel), $E_\gamma = 400$ MeV (bottom panel). Different values of p_B are obtained changing the scatterimg angles of the two outgoing protons. Positive (negative) values of p_B refer to situations where $\boldsymbol{p}_B$ is parallel (anti-parallel) to $\boldsymbol{q}$. Calculations are performed in the DW approach and with the TOF and Δ-parametrization as in Fig. 1. The dotted lines are the results of approach A [4], the solid lines are obtained with approach C.

In our previous calculations of two-nucleon knockout not all the CM effects were properly taken into account. In the CM frame the transition operator becomes a two-body operator even in the case of a one-body nuclear current. As a consequence, the one-body current can give a contribution to the cross section of two-particle emission independently of correlations. This effect is similar to the one of the effective charges in electromagnetic reactions [16].

The effective transition operator entering the transition matrix element is in principle defined consistently with the two-body initial and final state wave functions derived from an energy-dependent non-Hermitian Hamiltonian. In such an approach, no spurious contribution comes from the orthogonality defect of the wave functions [1, 9]. In practice, however, one approximates the transition operator in terms of simple forms of one- and two-body currents, thus introducing some spuriosity. In the past, this spuriousity was cured by subtracting from the transition amplitude the contribution of the one-body current without correlations in the nuclear wave functions. In this way, however, not only the spuriosity is subtracted, but also the CM effect given by the two-body operator which is present in the one-body current independently of correlations and which is not spurious. Alternatively, one can enforce orthogonality between the initial and final states by means of a Gram-Schmidt orthogonalization. The two approaches have been investigated here and shown to give similar results. However, the Gram-Schmidt orthogonalization has been further used in the present investigation because it is preferable in principle and allows us to naturally include all the CM effects.

The CM effects due to the one-body current without correlations are different in different situations and kinematics. For the ^{16}O(e,e′pp)^{14}C$_{\rm g.s.}$ reaction in the super-parallel kinematics these CM effects produce a strong enhancement of the contribution of the one-body current. As a consequence, the destructive interference between the one-body and the two-body Δ-current as well as the sensitivity to the treatment of the Δ-current discussed in [4] are strongly reduced. With respect to the results of [4], the calculated cross section is enhanced and seems to better reproduce the experimental data of [22]. On the other hand, these CM effects are very sensitive to the treatment of the two-nucleon overlap function describing the initial correlated pair of protons.

The mutual interaction between the two emerging protons produces a large enhancement of the cross section at medium and large recoil momenta. The effect is of the same type and of about the same relevance as in [8]. However, in contrast to the results of [8], where the enhancement at large p_B is due the Δ-current contribution, when including

CM effects it is essentially due to the one-body current, which is always dominant in the super-parallel cross section for all the values of the recoil momentum.

In the symmetrical kinematics the ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ reaction is dominated by the one-body current and is thus sensitive to the treatment of correlations, confirming the result found in [4]. In contrast, the ^{16}O(γ,pp)^{14}C$_{\rm g.s.}$ reaction is dominated by the Δ-current and is not affected by CM and orthogonalization effects up to recoil momenta of the order of 200 MeV/c.

In conclusion, the CM effects investigated in this work depend on kinematics and on the final state of the residual nucleus. The numerical examples shown in the present analysis indicate that these effects are particularly large for the ^{16}O(e,e$'$pp)^{14}C$_{\rm g.s.}$ reaction in the super-parallel kinematics of the MAMI experiment. The extreme sensitivity to the treatment of the different ingredients of the model and to different effects and contributions makes the super-parallel kinematics very interesting but also not particularly suitable to disentangle and investigate the specific contribution of short-range correlations. More and different situations should be considered to achieve this goal. Two examples have been shown in the symmetrical kinematics where either correlations or the Δ-current are dominant. In order to disentangle and investigate the different ingredients contributing to the cross sections, experimental data are needed in different kinematics which mutually supplement each other.

The investigation of CM effects and orthogonality between initial and final states will be extended in a forthcoming paper to the case of electromagnetic proton-neutron knockout, as urgently needed after the recent first measurements of the ^{16}O(e,e$'$pn)^{14}C$_{\rm g.s.}$ reaction performed at the MAMI microtron in Mainz [23].

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[1] S. Boffi, C. Giusti, F. D. Pacati, M. Radici, *Electromagnetic Response of Atomic Nuclei*, Oxford Studies in Nuclear Physics (Clarendon Press, Oxford, 1996); S. Boffi, C. Giusti, F. D. Pacati, Phys. Rep. **226**, 1 (1993).
[2] M. Anguiano, G. Co', A.M. Lallena, Nucl. Phys. A **744**, 168 (2004).
[3] J. Ryckebusch, W. van Nespen, Eur. Phys. J. A **20**, 435 (2004).
[4] C. Giusti, F. D. Pacati, M. Schwamb, S. Boffi, Eur. Phys. J. A **26**, 209 (2005).
[5] C. Giusti, F. D. Pacati, Nucl. Phys. A **535**, 573 (1991).
[6] C. Barbieri, C. Giusti, F. D. Pacati, W. H. Dickhoff, Phys. Rev. C **70**, 014606 (2004)
[7] M. Schwamb, S. Boffi, C. Giusti, F. D. Pacati, Eur. Phys. J. A **17**, 7 (2003).
[8] M. Schwamb, S. Boffi, C. Giusti, F. D. Pacati, Eur. Phys. J. A **20**, 233 (2004).
[9] S. Boffi, F. Cannata, F. Capuzzi, C. Giusti, F. D. Pacati, Nucl. Phys A **379**, 509 (1982).
[10] C. Giusti, F. D. Pacati, Nucl. Phys. A **615**, 373 (1997).
[11] R. D. Peccei, Phys. Rev. **176**, 1812 (1968); **181**, 1902 (1969).
[12] C. Giusti, F. D. Pacati, Nucl. Phys. A **641**, 297 (1998).
[13] P. Wilhelm, H. Arenhövel, C. Giusti, F.D. Pacati, Z. Phys. A **359**, 467 (1997).
[14] C. Giusti, F. D. Pacati, Phys. Rev. C **67**, 044601 (2003).
[15] D. F. Jackson, T. Berggren, Nucl. Phys. **62**, 353 (1965).
[16] S. Boffi, C. Giusti, F. D. Pacati, M. Rosa-Clot, Nuovo Cimento **76A**, 685 (1983).
[17] C. Giusti, F. D. Pacati, K. Allaart, W. J. W. Geurts, W. H. Dickhoff, H. M. Müther, Phys. Rev. C **57**, 1691 (1998).
[18] D. N. Kadrev, M. V. Ivanov, A. N. Antonov, C. Giusti, F. D. Pacati, Phys. Rev. C **68**, 014617 (2003).
[19] A. Nadasen *et al.,* Phys. Rev.C **23**, 1023 (1981).
[20] D. Knödler, H. Müther, P. Czerski, Phys. Rev. C **61**, 064603 (2000).
[21] S. Boffi, R. Cenni, C. Giusti, F. D. Pacati, Nucl. Phys A **420**, 38 (1984).
[22] G. Rosner, Prog. Part. Nucl. Phys. **44**, 99 (2000).
[23] D. G. Middleton *et al.,* Eur. Phys. J. A **29**, 261 (2006).
[24] R. Machleidt, K. Holinde, Ch. Elster, Phys. Rep. **149**, 1 (1987).
[25] C. C. Gearhart, Ph.D thesis, Washington University, St. Louis (1994);
C. C. Gearhart, W. H. Dickhoff, private communication.